Parametric resonance of a two-dimensional electron gas under bichromatic irradiation

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(Dated: February 6, 2008)

In an ultrahigh mobility 2D electron gas, even a weak nonparabolicity of the electron dispersion, by violating Kohn's theorem, can have a drastic effect on *dc* magnetotransport under *ac* drive. In this paper, we study theoretically the manifestation of this effect in the *dc* response to the combined action of *two* driving *ac*-fields (bichromatic irradiation). Compared to the case of monochromatic irradiation, which is currently intensively studied both experimentally and theoretically, the presence of a second microwave source provides additional insight into the properties of an *ac*-driven 2D electron gas in weak magnetic field. In particular, we find that nonparabolicity, being the simplest cause for a violation of Kohn's theorem, gives rise to new qualitative effects specific to bichromatic irradiation. Namely, when the frequencies ω_1 and ω_2 are well away from the cyclotron frequency, ω_c, our simple classical considerations demonstrate that the system becomes unstable with respect to fluctuations with frequency $\frac{1}{2}(\omega_1 + \omega_2)$. The most favorable condition for this *parametric* instability is $\frac{1}{2}(\omega_1 + \omega_2) \simeq \omega_c$. The saturation level of this instability is also determined by the nonparabolicity. We also demonstrate that, as an additional effect of nonparabolicity, this parametric instability can manifest itself in the *dc* properties of the system. This happens when ω_1, ω_2 and ω_c are related as $3:1:2$, respectively. Even for weak detuning between ω_1 and ω_2, the effect of the bichromatic irradiation on the *dc* response in the presence of nonparabolicity can differ dramatically from the monochromatic case. In particular, we demonstrate that, beyond a critical intensity of the two fields, the equations of motion acquire *multistable* solutions. As a result, the diagonal *dc*-conductivity can assume *several* stable negative values at the *same* magnetic field.

PACS numbers: 05.60.-k,73.43.Cd,73.50.Pz

I. INTRODUCTION

The cyclotron resonance in a 2D electron gas was first studied almost 30 years ago.[1,2] These studies revealed an oscillatory magnetoabsorption of microwave radiation with its principal peak at $\omega = \omega_c$, where ω_c and ω are the cyclotron and microwave frequencies, respectively, and several subharmonics at $\omega = n\omega_c$ due to the disorder-induced violation of Kohn's theorem.

Recently, interest in the properties of microwave-driven electrons in a magnetic field has been revived,[3] especially after experiments[4,5] carried out on samples with extremely high mobilities, indicated that near the cyclotron resonance and its harmonics, irradiation results in drastic changes of the diagonal *dc* resistivity. In contrast, the Hall resistivity remains practically unchanged by illumination, and retains its classical value. The experimental observations[4,5] were confirmed in a number of subsequent studies.[6,7,8,9,10,11,12,13,14] This unusual behavior of the weak-field magnetoconductivity is currently accounted for by an instability resulting from a sign reversal of the diagonal photoconductivity under irradiation.[15,16,17] The developed instability results in dynamical symmetry breaking,[15] i.e., in an inhomogeneous state of the system characterized by domains of current flowing in opposite directions. Ongoing theoretical studies[18,19,20,21,22,23,24] concentrate on the microscopic description of the sign reversal of photoconductivity. Closely related physics was already discussed theoretically quite long ago.[25,26]

Obviously, a complete understanding of the fascinating properties of ultraclean 2D electron systems under irradiation requires additional experimental studies. At the same time, the number of feasible measurements that were not carried out so far is limited. A promising avenue seems to be to study the response to *bichromatic* irradiation. Motivated by this, in the present paper we calculate this response within the simple model[27] of a clean *classical*[28] 2D gas, in which

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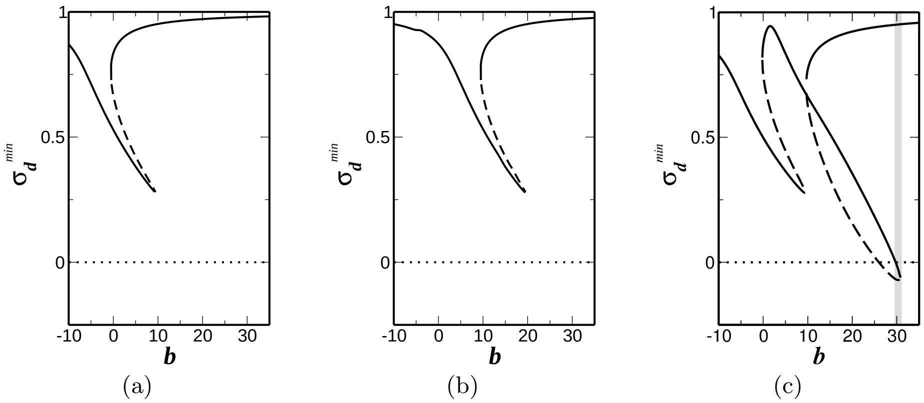

FIG. 1: Dimensionless diagonal conductivity (in units of the Drude value) plotted from Eq. (14) versus the dimensionless magnetic field, defined by Eq. (15), for three cases: (a) *monochromatic* irradiation with frequency ω_1 and dimensionless intensity $A = 14.4$; (b) *monochromatic* irradiation with the same intensity as in (a) and frequency $\omega_2 = 5\omega_1/3$; (c) *bichromatic* case: response to simultaneous irradiation with two microwave sources having the same intensities and frequencies as in (a) and (b). The emerging region of negative diagonal conductivity is shaded. All three plots (a)-(c) are calculated for $\frac{1}{2}(\omega_1 + \omega_2)\tau = 20$. Full and dashed lines correspond to stable and unstable branches, respectively.

Kohn's theorem is violated due to nonparabolicity

$$\varepsilon(p) = \frac{p^2}{2m}\left[1 - \frac{p^2}{2mE_0}\right], \tag{1}$$

of the electron dispersion. Here m is the effective mass, and E_0 is an energy of the order of the bandgap.

Denote by $\mathcal{E}_1$ and $\mathcal{E}_2$ the amplitudes of two linearly polarized *ac* fields with frequencies ω_1 and ω_2, respectively. In the presence of a *dc* field E_{dc}, the equation of motion for the electron momentum $\mathcal{P} = p_x + ip_y$ takes the form

$$\frac{d\mathcal{P}}{dt} + \frac{\mathcal{P}}{\tau} - i\omega_c\mathcal{P} + \frac{i\omega_c}{mE_0}\mathcal{P}|\mathcal{P}|^2 = eE_{dc}e^{i\theta} + \frac{e\mathcal{E}_1}{2}\Big(e^{i\omega_1 t} + e^{-i\omega_1 t}\Big) + \frac{e\mathcal{E}_2}{2}\Big(e^{i\omega_2 t} + e^{-i\omega_2 t}\Big), \tag{2}$$

where ω_c is the cyclotron frequency, τ is the relaxation time, and θ is the orientation of the weak *dc* field with respect to the fields $\mathcal{E}_1$, $\mathcal{E}_2$, which we assume to be parallel to each other. For a monochromatic *ac* drive, $\mathcal{E}_2 = 0$, it was demonstrated[27] that within a certain interval of magnetic fields near the cyclotron resonance, Eq. (2) yields negative diagonal conductivity, $\sigma_d < 0$, without significant change of the Hall conductivity. This sign reversal occurs when the mobility is high, $\omega_c\tau \gg 1$, and $\mathcal{E}_1$ is sufficiently strong. In the vicinity of the cyclotron resonance, σ_d turns negative even when the irradiation-induced change of the electron mass is relatively weak. Thus, the simple model Eq. (2) exhibits negative photoconductivity without invoking Landau quantization. It also predicts bistable hysteretic behavior of σ_d as a function of the detuning from the cyclotron resonance for large enough $\omega_c\tau$.

In the present paper, we extend the consideration of Ref. 27 to the bichromatic case. The most convincing illustration that the response to irradiation with two *ac*-fields cannot simply be reduced to the superposition of the responses to each individual field, is presented in Fig. 1. It is seen in Figs. 1(a,b) that the individual fields of equal intensity and frequency ratio 5 : 3 are unable to reverse the sign of the diagonal conductivity at any magnetic field. At the same time, upon *simultaneous* irradiation by the both fields, a domain of magnetic fields emerges, within which the diagonal conductivity is negative (see Fig. 1(c)).

In addition, our study reveals the following new features that are specific to the bichromatic case:

(*i*) The presence of the second *ac* field on the r.h.s. of Eq. (2) gives rise to a second domain of magnetic field, within which σ_d is negative. Upon increasing the intensities of the two *ac* fields, the two domains of negative photoconductivity merge into a single domain which broadens much faster with the *ac* intensity than in the monochromatic case.

(*ii*) For monochromatic irradiation, σ_d could assume either one or two stable values. By contrast, under bichromatic irradiation, we find a *multistable* regime within certain domains of magnetic field.

(*iii*) In the vicinity of the conditions $(\omega_1 + \omega_2) = 2\omega_c$ and $|\omega_1 - \omega_2| = 2\omega_c$, a nonparabolicity-induced *parametric* instability develops in the system. As a result of this instability, the components $(\omega_1 + \omega_2)/2$ and $|\omega_1 - \omega_2|/2$ emerge in addition to the conventional frequencies ω_1 and ω_2 of the momentum oscillations. These components, in turn, upon mixing with the components ω_1, ω_2, give rise to components of $\mathcal{P}$ oscillating with frequencies $3\omega_1 - \omega_2$. Thus, for *bichromatic* irradiation, two high-frequency *ac* driving fields can create a low frequency current circulating in the system. In particular, for $\omega_2 = 3\omega_1 \approx 3\omega_c/2$ the system exhibits a *dc* response to the *ac* drive.

The paper is organized as follows. The case of weak detuning from the cyclotron frequency is treated analytically in Sec. II. In the same section, we present numerical results in this limit, which exhibit nontrivial multistable behavior. In Sec. III, we consider the case of strong detuning, where we find a nonparabolicity-induced parametric resonance. Concluding remarks are presented in Sec. IV.

II. WEAK DETUNING

For monochromatic irradiation, the cyclotron resonance develops when the microwave frequency is close to the cyclotron frequency ω_c. In this section, we consider bichromatic irradiation when both frequencies ω_1 and ω_2 are close to ω_c, $|\omega_1 - \omega_c| \ll \omega_c$ and $|\omega_2 - \omega_c| \ll \omega_c$, so that the cyclotron resonances due to ω_1 and ω_2 can interfere with one another.

A. Calculation of diagonal conductivity

In analogy to Ref. 27, we search for solutions of Eq. (2) in the form

$$\mathcal{P}(t) = \mathcal{P}_0 + \mathcal{P}_1^+ \exp(i\omega_1 t) + \mathcal{P}_1^- \exp(-i\omega_1 t) + \mathcal{P}_2^+ \exp(i\omega_2 t) + \mathcal{P}_2^- \exp(-i\omega_2 t), \tag{3}$$

where $\mathcal{P}_0$ is a small dc component proportional to E_{dc}. The components $\mathcal{P}_1^-$ and $\mathcal{P}_2^-$ are nonresonant and can be found from the simplified equations

$$-\, i(\omega_1 + \omega_c)\mathcal{P}_1^- = \frac{e\mathcal{E}_1}{2}, \tag{4}$$

$$-\, i(\omega_2 + \omega_c)\mathcal{P}_2^- = \frac{e\mathcal{E}_2}{2}, \tag{5}$$

where we neglect both relaxation and nonlinearity. However, relaxation and nonlinearity must be taken into account when calculating the resonant components $\mathcal{P}_1^+$ and $\mathcal{P}_2^+$. Substituting Eq. (3) into Eq. (2), and taking into account that $|\mathcal{P}_1^-|, |\mathcal{P}_2^-| \ll |\mathcal{P}_1^+|, |\mathcal{P}_2^+|$, we arrive at a system of coupled equations for the resonant momentum components,

$$\left[i\left(\omega_1 - \omega_c\right) + \frac{1}{\tau} + \frac{i\omega_c}{mE_0}\left(|\mathcal{P}_1^+|^2 + 2|\mathcal{P}_2^+|^2\right)\right]\mathcal{P}_1^+ = \frac{e\mathcal{E}_1}{2}, \tag{6}$$

$$\left[i\left(\omega_2 - \omega_c\right) + \frac{1}{\tau} + \frac{i\omega_c}{mE_0}\left(2|\mathcal{P}_1^+|^2 + |\mathcal{P}_2^+|^2\right)\right]\mathcal{P}_2^+ = \frac{e\mathcal{E}_2}{2}. \tag{7}$$

Despite the inequalities $|\mathcal{P}_1^+| \ll |\mathcal{P}_1^-|$ and $|\mathcal{P}_2^+| \ll |\mathcal{P}_2^-|$, it is crucial to keep the nonresonant components $\mathcal{P}_1^-$ and $\mathcal{P}_2^-$ when considering the dc component $\mathcal{P}_0$. This yields

$$\left[-i\omega_c + \frac{1}{\tau} + \frac{2i\omega_c}{mE_0}\left(|\mathcal{P}_1^+|^2 + |\mathcal{P}_2^+|^2\right)\right]\mathcal{P}_0 + \frac{2i\omega_c}{mE_0}\left[\mathcal{P}_1^+\mathcal{P}_1^- + \mathcal{P}_2^+\mathcal{P}_2^-\right]\mathcal{P}_0^* = eE_{dc}e^{i\theta}. \tag{8}$$

Due to the nonlinearity, the microwave intensities induce an effective shift in the resonance frequency ω_c. Thus, it is convenient to introduce effective detunings Ω_1 and Ω_2 by

$$\Omega_1 = \omega_1 - \omega_c + \frac{\omega_c}{mE_0}\left(|\mathcal{P}_1^+|^2 + 2|\mathcal{P}_2^+|^2\right), \tag{9}$$

$$\Omega_2 = \omega_2 - \omega_c + \frac{\omega_c}{mE_0}\left(2|\mathcal{P}_1^+|^2 + |\mathcal{P}_2^+|^2\right), \tag{10}$$

and to present formal solutions of Eqs. (6) and (7) in the form

$$\mathcal{P}_1^+ = \frac{e\mathcal{E}_1\tau}{2\left(1 + i\Omega_1\tau\right)} \quad , \qquad \mathcal{P}_2^+ = \frac{e\mathcal{E}_2\tau}{2\left(1 + i\Omega_2\tau\right)}. \tag{11}$$

Note that the detunings Ω_1 and Ω_2 themselves depend on $\mathcal{P}_1^+$ and $\mathcal{P}_2^+$, so that Eqs. (9), (10) and (11) should be considered as a system of nonlinear equations for the resonant momentum components $\mathcal{P}_1^+$ and $\mathcal{P}_2^+$. Assuming that the detunings Ω_1 and Ω_2 are known, the solution of Eq. (8) yields for the *dc* component

$$\mathcal{P}_0 = \frac{eE_{dc}}{\omega_c^2\tau}\left[(1+i\omega_c\tau)\,e^{i\theta} + \frac{1}{4mE_0}\left\{\frac{(e\mathcal{E}_1\tau)^2}{1+i\Omega_1\tau} + \frac{(e\mathcal{E}_2\tau)^2}{1+i\Omega_2\tau}\right\}e^{-i\theta}\right]. \tag{12}$$

The diagonal conductivity σ_d is proportional to $\mathrm{Re}\left(\mathcal{P}_0 e^{-i\theta}\right)$. Thus, the second term in Eq. (12) gives rise to a θ-dependence of the nonparabolicity-induced contribution to the diagonal conductivity which is given by $\sin(2\theta-\phi)$. Here, ϕ satisfies the equation

$$\tan\phi = \frac{\Omega_1\tau\left(1+\Omega_2^2\tau^2\right)\mathcal{E}_1^2 + \Omega_2\tau\left(1+\Omega_1^2\tau^2\right)\mathcal{E}_2^2}{\left(1+\Omega_2^2\tau^2\right)\mathcal{E}_1^2 + \left(1+\Omega_1^2\tau^2\right)\mathcal{E}_2^2}. \tag{13}$$

We deduce that the minimal value of σ_d is given by

$$\sigma_d^{min} = \frac{ne^2}{m\omega_c^2\tau}\left\{1 - \frac{e^2\tau^2}{4mE_0}\left[\frac{\left(\mathcal{E}_1^2\Omega_2\tau + \mathcal{E}_2^2\Omega_1\tau\right)^2 + \left(\mathcal{E}_1^2+\mathcal{E}_2^2\right)^2}{\left(1+\Omega_1^2\tau^2\right)\left(1+\Omega_2^2\tau^2\right)}\right]^{1/2}\right\}. \tag{14}$$

In the following, we will be particularly interested in σ_d^{min}, since the condition $\sigma_d^{min} < 0$ is sufficient for the formation of the zero-resistance state.

B. Numerical results: Multistability

As demonstrated in Ref. 27, the diagonal conductivity in the monochromatic case shows a region of bistability. In the bichromatic case under study, there can even be multistable behavior as will now be shown. We measure the frequency difference of the *ac*-fields by $\Delta = (\omega_1 - \omega_2)\,\tau$ and the magnetic field by

$$b = \left(\omega_c - \frac{\omega_1+\omega_2}{2}\right)\tau, \tag{15}$$

which depends linearly on the magnetic field B. Upon substituting the formal solutions $\mathcal{P}_1^+$ and $\mathcal{P}_2^+$ of Eq. (11) into Eqs. (9-10), these can be written as a pair of coupled equations for the effective detunings $\Omega_1\tau$ and $\Omega_2\tau$

$$\Omega_1\tau = \frac{\Delta}{2} - b + A\left[\frac{1}{1+(\Omega_1\tau)^2} + \frac{2\eta^2}{1+(\Omega_2\tau)^2}\right], \qquad \Omega_2\tau = -\frac{\Delta}{2} - b + A\left[\frac{2}{1+(\Omega_1\tau)^2} + \frac{\eta^2}{1+(\Omega_2\tau)^2}\right], \tag{16}$$

where η and A, given by

$$\eta = \mathcal{E}_2/\mathcal{E}_1, \qquad A = \omega_c\tau\frac{(e\mathcal{E}_1\tau)^2}{4mE_0}, \tag{17}$$

measure the ratio of the field amplitudes and the ratio of the absolute field intensities to the nonparabolicity of the electron spectrum, respectively. As in the monochromatic case, the strength of the first order correction to the Drude conductivity is proportional to the microwave intensity and thus A. At a fixed magnetic field b and at fixed *ac* frequencies and amplitudes, this coupled system of two third-order equations can yield up to nine simultaneous solutions $(\Omega_1\tau, \Omega_2\tau)$ for the effective detunings. Since σ_d^{min} is directly related to these effective detunings via

$$\sigma_d^{min} = \sigma_D\left\{1 - \frac{A}{\omega_c\tau}\left[\frac{1}{1+(\Omega_1\tau)^2} + \frac{\eta^4}{1+(\Omega_2\tau)^2} + \frac{2\eta^2\left(1+\Omega_1\tau\Omega_2\tau\right)}{\left(1+(\Omega_1\tau)^2\right)\left(1+(\Omega_2\tau)^2\right)}\right]^{1/2}\right\}, \tag{18}$$

where $\sigma_D = ne^2/(\omega_c^2\tau)$ is the Drude conductivity, there are thus up to nine individual branches of σ_d^{min} at a given b. This multistable behavior occurs in the vicinity of the resonance and will be studied below for the specific case of $\eta = 1$.

We first focus on the dependence of $\sigma_d^{min}(b)$ on Δ. For large Δ, i.e. markedly different *ac*-frequencies, we expect two separate regions in b where σ_d^{min} deviates significantly from the Drude result. These are the regions where the

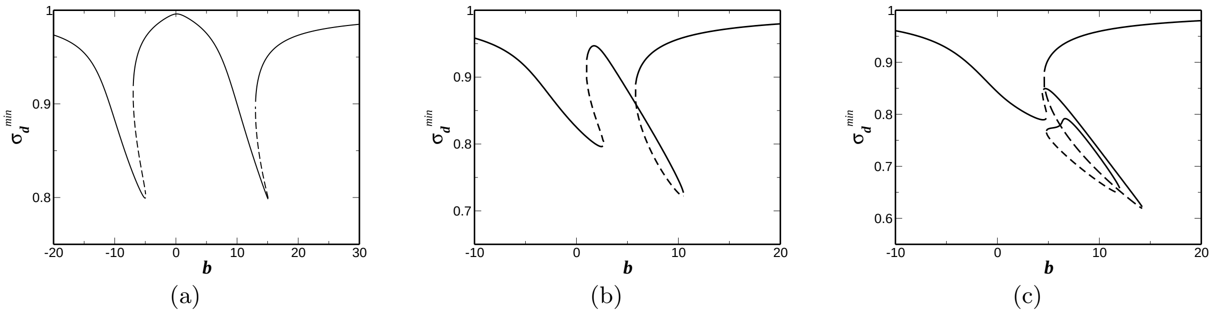

FIG. 2: Evolution of the dimensionless (in units of the Drude conductivity σ_D) minimal conductivity σ_d^{min} as function of magnetic field b, defined in Eq. (15), for three different values of Δ; (a) $\Delta = 20$, (b) $\Delta = 5$, (c) $\Delta = 1$. The curves are calculated for the values of parameters $\eta = 1$, $A = 5$ (defined by Eq. (17)), and $\omega_c\tau = 25$. The distance of the two dips that can be clearly discerned in (a) is roughly Δ. When lowering Δ, the dips move closer together (b) and finally merge (c). In addition, multistable regions emerge. As in Fig. 1, unstable branches are plotted as dashed lines.

cyclotron frequency is in resonance with one of the two *ac* frequencies, i.e. either $\omega_1 \simeq \omega_c$ or $\omega_2 \simeq \omega_c$. Inside these regions, the behavior with respect to b is very similar to the monochromatic case, except that the irradiation-induced effective shift of ω_c now depends on both external frequencies. In particular, the emergence of bistable regions inside these two separate intervals as in the monochromatic case is to be expected. This can be seen in Fig. 2(a), where σ_d^{min} is shown as a function of magnetic field b for rather large Δ. Two dips in σ_d^{min} can be clearly discerned, the inner branches of which are unstable. Upon reducing Δ, the two dips move closer together up to a point where the frequencies ω_1 and ω_2 are so close that the analogy to the monochromatic case breaks down and the two dips start to interact to finally from a single multistable dip in the limit $\Delta \to 0$. This behavior is exemplified in Figs. 2(b,c). It can be seen that multiple solutions of $\sigma_d^{min}(b)$ develop upon reducing Δ.

Next, we consider the case of negative diagonal conductivity and study the evolution of σ_d^{min} with magnetic field. As expected, there is a threshold intensity below which no negative diagonal conductivity is observed. Upon increasing the field amplitudes and thus A, the negative first order correction to the Drude conductivity grows linearly with $A/(\omega_c\tau)$ as can be seen from Eq. (18). When this correction exceeds one, negative σ_d^{min} is to be expected in some regions of magnetic field b. Fig. 3 shows the b-dependence of σ_d^{min} for three specific values of A. It can be seen that at low A, no regions in b with negative diagonal conductivity can be observed. At higher A, two regions in b show negative σ_d^{min}-branches as is also indicated by the shaded regions in Fig. 3(b). For even higher A, a single large region in b shows negative diagonal conductivity.

To clarify the evolution of these regions with increasing field amplitudes, we plotted the extension of the regions in b as a function of A. The result is shown in Fig. 4. It is remarkable that above the threshold value of A, first a single region appears that shows negative σ_d^{min}. Then, in the immediate vicinity of the threshold a second, well separated

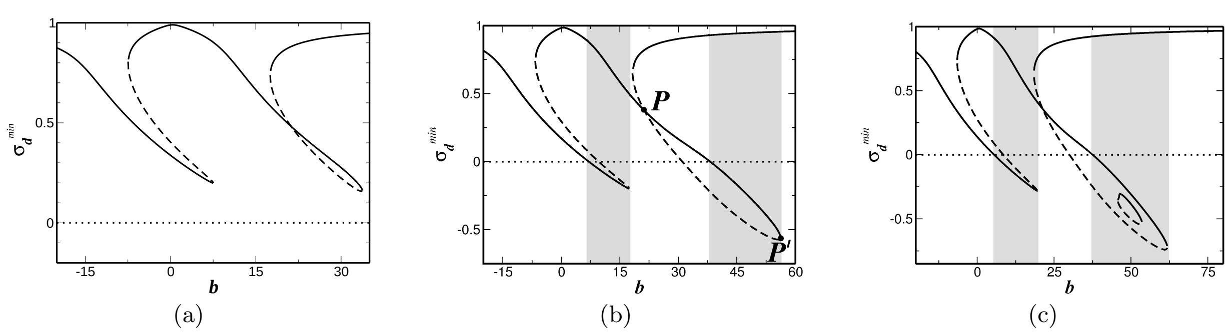

FIG. 3: The dimensionless minimal diagonal conductivity, $\sigma_d^{min}(b)$, defined by Eq. (18), is plotted for three different values of the dimensionless intensity A of the two *ac*-fields; (a) $A = 20$, (b) $A = 30$, (c) $A = 32$. The domains of magnetic field b with negative σ_d^{min} are shaded. The dotted line indicates the boundary between positive and negative σ_d^{min}. Unstable branches are dashed as in Fig. 1. The curves are calculated for the values of the parameters $\eta = 1$, $\Delta = (\omega_1 - \omega_2)\tau = 25$ and $\omega_c\tau = 25$. In (b), the point $\mathbf{P}$ is a point where a continuous stable and a continuous unstable branch intersect without "noticing" each other, while the point $\mathbf{P'}$ is a cusp which separates a stable from an unstable branch.

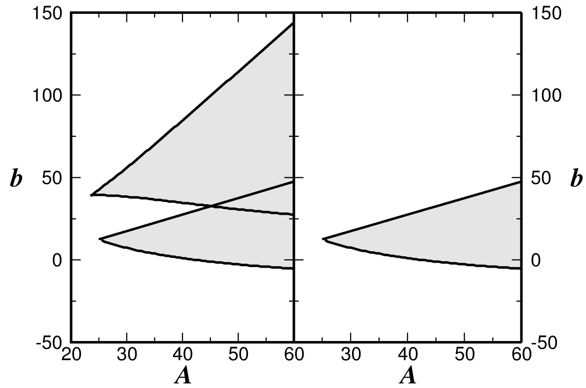

FIG. 4: Evolution of the regions of negative σ_d^{min} with irradiation intensity, A. Shown are the bichromatic case (left panel) and the monochromatic case (right panel). In both cases, only the dominant (stable) branches are shown to avoid confusion. The parameters are the same as in Fig. 3.

region develops. Upon further increasing A, the width of these regions grows and, eventually, the two regions merge to form a single broad region of negative diagonal conductivity. For comparison, we also show the monochromatic case in the right hand panel of Fig. 4.

C. Stability of different branches

The stability of the various branches of $\sigma_d^{min}(b)$ as shown by solid and dashed lines in Figs. 1,2, and 3, can be obtained from a standard stability analysis. As usual, we find that stable and unstable branches "meet" at cusps, as clearly seen at the minima of σ_d in Figs. 1 and 2. The transitions from unstable to stable branches at larger b in these figures are also accompanied by cusps, although this can not necessarily be discerned within the resolution of the figures. The number of branches increases with the irradiation intensity, cf. Figs. 2 and 3. The rule that stable and unstable branches meet in cusps remains valid, although this statement becomes less trivial. For example, in Fig. 3(b) stable and unstable branches intersect at point $\mathbf{P}$ without "noticing each other". Accordingly, there is no cusp at this point. At the same time, there is a cusp at $\mathbf{P}'$ in Fig. 3(b) where the same branches switch between stable and unstable. Figs. 2(c) and 3(c) illustrate how new branches and multistability emerge with increasing irradiation intensity. The emergence of new stable and unstable branches occurs in pairs which meet at additional cusps. In both figures 2(c) and 3(c), there are regions in magnetic field with three coexisting stable solutions (tristability). Further increase of A would lead to up to eight cusps in Fig. 3(c), each of which is a meeting point of stable and unstable branches. Thus, the tristability situation illustrated in Fig. 3(c) will evolve into a magnetic field domain with "four-stability".

III. STRONG DETUNING

In this section, we consider the case when both frequencies ω_1 and ω_2 are tuned away from ω_c. This implies that the system (6-7) decouples and acquires the obvious solutions

$$\mathcal{P}_1^+ = \frac{e\mathcal{E}_1\tau}{2\left[1+i\left(\omega_1-\omega_c\right)\tau\right]}, \qquad \mathcal{P}_2^+ = \frac{e\mathcal{E}_2\tau}{2\left[1+i\left(\omega_2-\omega_c\right)\tau\right]}. \tag{19}$$

The condition $e\mathcal{E}_1$, $e\mathcal{E}_2 \ll \omega_c\left(mE_0\right)^{1/2}$ for decoupling follows from Eqs. (6-7), assuming that $|\omega_1-\omega_c|$, $|\omega_2-\omega_c|$ $\sim \omega_c \gg 1/\tau$. Interestingly, even under this condition, the solutions are unstable for certain relations between the frequencies ω_1, ω_2. The mechanism for this instability relies on mixing of the two external drive frequencies by the nonparabolicity which results in a modulation of the effective cyclotron frequency. This modulation, in turn, can lead to *parametric* resonance.

To perform the stability analysis of the solutions Eq. (19), we introduce a small deviation $\mathcal{P} \to \mathcal{P} + \delta\mathcal{P}$ and linearize

Eq. (2) with respect to $\delta\mathcal{P}$. The linearized equation (2) has the form

$$\frac{d}{dt}(\delta\mathcal{P})+\left(\frac{1}{\tau}-i\omega_c+\frac{2i\omega_c}{mE_0}|\mathcal{P}|^2\right)\delta\mathcal{P}+\frac{i\omega_c}{mE_0}\mathcal{P}^2(\delta\mathcal{P})^*=0. \tag{20}$$

This equation couples $\delta\mathcal{P}$ to $(\delta\mathcal{P})^*$ via the nonparabolicity of the electron spectrum. The corresponding equation for $\delta\mathcal{P}^*$ reads

$$\frac{d}{dt}(\delta\mathcal{P}^*)+\left(\frac{1}{\tau}+i\omega_c-\frac{2i\omega_c}{mE_0}|\mathcal{P}|^2\right)\delta\mathcal{P}^*-\frac{i\omega_c}{mE_0}(\mathcal{P}^*)^2\,\delta\mathcal{P}=0. \tag{21}$$

The coupling coefficient, $\mathcal{P}^2$, as seen from Eq. (3), contains the harmonics $\pm 2\omega_1$, $\pm 2\omega_2$, $\pm(\omega_1+\omega_2)$, and $\pm(\omega_1-\omega_2)$. This suggests that $\delta\mathcal{P}(t)$ also contains a number of harmonics, namely, $\pm\omega_1$, $\pm\omega_2$, $\pm(\omega_1+\omega_2)/2$, and $\pm(\omega_1-\omega_2)/2$. An instability might develop when one of these frequencies is close to ω_c. Thus, in the monochromatic case, the instability develops only in the vicinity of the cyclotron resonance $\omega_1\approx\omega_c$. The branches, shown with dashed lines, in Figs. 1(a,b), are unstable due to this instability. By contrast, the bichromatic case offers two additional options for an instability to develop, even if the frequencies ω_1, ω_2 are nonresonant, namely $\omega_c\approx(\omega_1+\omega_2)/2$ and $\omega_c\approx|(\omega_1-\omega_2)|/2$. The considerations of both cases are analogous to each other. Therefore, we focus on the first case below.

A. Parametric instability at $(\omega_1+\omega_2)\approx 2\omega_c$

Upon substituting the ansatz

$$\delta\mathcal{P}(t)=C\exp\left\{\left[\Gamma+\frac{i(\omega_1+\omega_2)}{2}\right]t\right\},\qquad \delta\mathcal{P}(t)^*=C^*\exp\left\{\left[\Gamma-\frac{i(\omega_1+\omega_2)}{2}\right]t\right\} \tag{22}$$

into Eqs. (20-21) and keeping only resonant terms, we obtain the following system of algebraic equations for C and C^*

$$\left[\Gamma+\frac{1}{\tau}+\frac{i(\omega_1+\omega_2-2\omega_c)}{2}+\frac{2i\omega_c}{mE_0}\left(|\mathcal{P}_1^+|^2+|\mathcal{P}_2^+|^2\right)\right]C = -\frac{2i\omega_c}{mE_0}\mathcal{P}_1^+\mathcal{P}_2^+C^* \tag{23}$$

$$\left[\Gamma+\frac{1}{\tau}-\frac{i(\omega_1+\omega_2-2\omega_c)}{2}-\frac{2i\omega_c}{mE_0}\left(|\mathcal{P}_1^+|^2+|\mathcal{P}_2^+|^2\right)\right]C^* = \frac{2i\omega_c}{mE_0}\left(\mathcal{P}_1^+\mathcal{P}_2^+\right)^*C. \tag{24}$$

Thus, the most favorable condition for instability is determined by the following relation between ω_1 and ω_2

$$\begin{aligned}\omega_1+\omega_2 &= 2\omega_c\left[1-\frac{2}{mE_0}\left(|\mathcal{P}_1^+|^2+|\mathcal{P}_2^+|^2\right)\right]\\ &\approx 2\omega_c\left\{1-\frac{e^2}{2mE_0}\left[\frac{\mathcal{E}_1^2}{(\omega_1-\omega_c)^2}+\frac{\mathcal{E}_2^2}{(\omega_2-\omega_c)^2}\right]\right\}\approx 2\omega_c\left[1-\frac{2e^2\left(\mathcal{E}_1^2+\mathcal{E}_2^2\right)}{mE_0(\omega_1-\omega_2)^2}\right],\end{aligned} \tag{25}$$

where Eq. (19) has been used. In this case, the increment Γ is maximal and given by

$$\Gamma_{max}=-\frac{1}{\tau}+\left(\frac{2\omega_c}{mE_0}\right)|\mathcal{P}_1^+\mathcal{P}_2^+|\approx-\frac{1}{\tau}+\left|\frac{e^2\omega_c\mathcal{E}_1\mathcal{E}_2}{mE_0(\omega_1-\omega_c)(\omega_2-\omega_c)}\right|\approx-\frac{1}{\tau}+\frac{e^2(\omega_1+\omega_2)|\mathcal{E}_1\mathcal{E}_2|}{mE_0(\omega_1-\omega_2)^2}. \tag{26}$$

The parametric instability develops if Γ_{max} is positive. It is important to note that the condition $\Gamma_{max}>0$ is consistent with the condition of strong detuning when the simplified expressions Eq. (19) are valid. Indeed, assuming $|\omega_1-\omega_c|\sim|\omega_2-\omega_c|\sim\omega_c$, the two conditions can be presented as $\omega_c(mE_0)^{1/2}\gg e\mathcal{E}_1, e\mathcal{E}_2\gg\tau^{-1}(mE_0)^{1/2}$. Therefore, for $\omega_c\tau\gg 1$, there exists an interval of the amplitudes of the *ac* fields within which both conditions are met. Note also, that parametric resonance does not develop exactly at $\omega_c=(\omega_1+\omega_2)/2$, i.e. at $b=0$ (in dimensionless units, see Eq. (15)). In fact, from Eqs. (25) and (26) it can be concluded that $\Gamma_{max}>0$ corresponds to $b\gtrsim 1$. In experimental situations, when ω_1 and ω_2 are fixed, Eq. (25) can also be viewed as an expression for the magnetic field $\omega_c=\omega_c^{opt}$,

at which the parametric instability is most pronounced. The interval of ω_c around ω_c^{opt}, within which the increment is positive can be found from the dependence $\Gamma(\omega_c)$

$$\Gamma(\omega_c) = -\frac{1}{\tau} + \sqrt{\left(\Gamma_{max} + \frac{1}{\tau}\right)^2 - \left(\omega_c - \omega_c^{opt}\right)^2} \approx -\frac{1}{\tau} + \sqrt{\left[\frac{e^2\left(\omega_1+\omega_2\right)|\mathcal{E}_1\mathcal{E}_2|}{mE_0\left(\omega_1-\omega_2\right)^2}\right]^2 - \left(\omega_c - \omega_c^{opt}\right)^2}. \tag{27}$$

Upon setting $\Gamma(\omega_c) = 0$ in the l.h.s. of Eq. (27), we find the width of the interval to be

$$|\omega_c - \omega_c^{opt}| \le \left\{\left[\frac{e^2\omega_c|\mathcal{E}_1\mathcal{E}_2|}{2mE_0\left(\omega_1-\omega_c\right)^2}\right]^2 - \frac{1}{\tau^2}\right\}^{1/2} \approx \left\{\left[\frac{e^2\left(\omega_1+\omega_2\right)|\mathcal{E}_1\mathcal{E}_2|}{mE_0\left(\omega_1-\omega_2\right)^2}\right]^2 - \frac{1}{\tau^2}\right\}^{1/2}. \tag{28}$$

It is instructive to reformulate the condition for the parametric instability in a different way. Assume for simplicity that $\mathcal{E}_1 = \mathcal{E}_2$. Then the combination $e^2|\mathcal{E}_1\mathcal{E}_2|/2mE_0(\omega_1-\omega_c)^2$ is equal to $\delta m/m$, where $\delta m/m$ is the relative correction to the electron effective mass due to irradiation.[27] From Eq. (26) it follows that the condition $\Gamma_{max} > 0$ can be presented as $(\omega_c\tau)(\delta m/m) > 1$. With $\omega_c\tau \gg 1$ this condition can be satisfied even for $\delta m \ll m$. Note, that in the case of weak detuning, the same condition is required for the *dc* conductivity to assume negative values near the cyclotron resonance.

Summarizing, we arrive at the following scenario. In the case of strong detuning, there is no mutual influence of the responses to the *ac* fields $\mathcal{E}_1$ and $\mathcal{E}_2$ as long as they are weak. However, as the product $|\mathcal{E}_1\mathcal{E}_2|$ increases and reaches a critical value $|\mathcal{E}_1\mathcal{E}_2|_c$, the threshold, $\Gamma_{max} = 0$, where Γ_{max} is given by Eq. (26), is exceeded at the magnetic field $\omega_c = \omega_c^{opt}$ determined by Eq. (25). Above the threshold, fluctuations with frequencies close to $(\omega_1+\omega_2)/2$ are amplified. This effect of parametric instability is solely due to nonparabolicity. Then the natural question arises: At what level does the component of momentum with frequency $(\omega_1+\omega_2)/2$ saturate above threshold? This question is addressed in the next subsection.

B. Parametric instability at $|\omega_1 - \omega_2| \approx 2\omega_c$

We now briefly discuss parametric instability at weak magnetic field, $\omega_c \approx |(\omega_1-\omega_2)/2|$. Assume for concreteness, that $\omega_1 > \omega_2$. In this case, the optimal magnetic field, $\tilde{\omega}_c^{opt}$, is lower and reads

$$\omega_1 - \omega_2 \approx 2\tilde{\omega}_c^{opt}\left\{1 - \frac{e^2}{2mE_0}\left[\frac{\mathcal{E}_1^2}{\left(\omega_1-\tilde{\omega}_c^{opt}\right)^2} + \frac{\mathcal{E}_2^2}{\left(\omega_2-\tilde{\omega}_c^{opt}\right)^2}\right]\right\} \approx 2\tilde{\omega}_c^{opt}\left\{1 - \frac{2e^2}{mE_0}\left[\frac{\mathcal{E}_1^2}{\left(\omega_1+\omega_2\right)^2} + \frac{\mathcal{E}_2^2}{\left(3\omega_2-\omega_1\right)^2}\right]\right\}. \tag{29}$$

At $\omega_c = \tilde{\omega}_c^{opt}$, the threshold condition for parametric instability, analogous to Eq. (26), has the form

$$\tilde{\Gamma}_{max} \approx -\frac{1}{\tau} + \frac{e^2\left(\omega_1-\omega_2\right)|\mathcal{E}_1\mathcal{E}_2|}{mE_0\left(\omega_1+\omega_2\right)|3\omega_2-\omega_1|} > 0. \tag{30}$$

There is no real divergence in Eqs. (29), (30) in the limit $\omega_1 \to 3\omega_2$, since these are derived under the assumption that the difference $|3\omega_2 - \omega_1|$ is $\gtrsim 1/\tau$.

C. Saturation of parametric resonance

As the threshold for parametric resonance is exceeded, the harmonics with frequency $(\omega_1+\omega_2)/2$ can no longer be considered as a perturbation, but rather have to be included into the equation of motion. In other words, we must search for a solution of Eq. (2) in the form

$$\mathcal{P} = \mathcal{P}_1^+\exp\left(i\omega_1 t\right) + \mathcal{P}_2^+\exp\left(i\omega_2 t\right) + \mathcal{P}_3(t)\exp\left[i\left(\frac{\omega_1+\omega_2}{2}\right)t\right], \tag{31}$$

where $\mathcal{P}_3(t)$ is a slowly varying function of time. Upon substituting this form into Eq. (2), we obtain the following coupled equations for $\mathcal{P}_3(t)$ and $\mathcal{P}_3^*(t)$

$$\frac{d\mathcal{P}_3}{dt} + \left[\frac{1}{\tau} + \frac{i\left(\omega_1+\omega_2-2\omega_c\right)}{2} + \frac{i\omega_c}{mE_0}\left(2|\mathcal{P}_1^+|^2 + 2|\mathcal{P}_2^+|^2 + |\mathcal{P}_3|^2\right)\right]\mathcal{P}_3 = -\frac{2i\omega_c}{mE_0}\mathcal{P}_1^+\mathcal{P}_2^+\mathcal{P}_3^*, \tag{32}$$

$$\frac{d\mathcal{P}_3^*}{dt}+\left[\frac{1}{\tau}-\frac{i\left(\omega_1+\omega_2-2\omega_c\right)}{2}-\frac{i\omega_c}{mE_0}\left(2|\mathcal{P}_1^+|^2+2|\mathcal{P}_2^+|^2+|\mathcal{P}_3|^2\right)\right]\mathcal{P}_3^*=\frac{2i\omega_c}{mE_0}\left(\mathcal{P}_1^+\mathcal{P}_2^+\right)^*\mathcal{P}_3. \tag{33}$$

Saturated parametric instability is described by setting $d\mathcal{P}_3/dt=0$ and $d\mathcal{P}_3^*/dt=0$ in Eqs. (32) and (33), respectively. The result for $\mathcal{P}_3$ has the simplest form for the optimal magnetic field $\omega_c=\omega_c^{opt}$

$$|\mathcal{P}_3|(\omega_c^{opt})=\left[|\mathcal{P}_1^+\mathcal{P}_2^+|^2-\frac{4m^2E_0^2}{\left(\omega_1+\omega_2\right)^2\tau^2}\right]^{1/4}. \tag{34}$$

From Eq. (34) we conclude that, in the vicinity of the threshold, $|\mathcal{P}_3|$ increases as $\Big(|\mathcal{E}_1\mathcal{E}_2|-|\mathcal{E}_1\mathcal{E}_2|_c\Big)^{1/4}\propto\Gamma_{max}^{1/4}$. Well above the threshold it approaches the value $\sqrt{|\mathcal{P}_1^+\mathcal{P}_2^+|}$. From here we conclude that, even upon saturation, the magnitude of the nonparabolicity-induced harmonics with frequency $\left(\omega_1+\omega_2\right)/2$ does not have a "back" effect on the magnitudes Eq. (19) of the responses to the *ac* fields.

For magnetic fields in the vicinity of ω_c^{opt} the saturation value, $|\mathcal{P}_3|(\omega_c)$, is given by

$$|\mathcal{P}_3|(\omega_c)=\left\{\left[|\mathcal{P}_1^+\mathcal{P}_2^+|^2-\frac{4m^2E_0^2}{\left(\omega_1+\omega_2\right)^2\tau^2}\right]^{1/2}-\frac{2mE_0|\omega_c-\omega_c^{opt}|}{\left(\omega_1+\omega_2\right)}\right\}^{1/2}\simeq\left[|\mathcal{P}_3|^2(\omega_c^{opt})-\frac{2mE_0|\omega_c-\omega_c^{opt}|}{\left(\omega_1+\omega_2\right)}\right]^{1/2}. \tag{35}$$

In contrast to $|\mathcal{P}_3|(\omega_c^{opt})$, the threshold behavior of $|\mathcal{P}_3|(\omega_c)$ is slower, namely $|\mathcal{P}_3|(\omega_c)\propto\Big(|\mathcal{E}_1\mathcal{E}_2|-|\mathcal{E}_1\mathcal{E}_2|_c\Big)^{1/2}$. In principle, one has to verify that the solutions Eqs. (34), (35), that describe the saturated parametric resonance, are stable. This can be done with the use of the system Eqs. (32), (33), by perturbing it around the saturated solution. The outcome of this consideration is that the corresponding perturbations do indeed decay.

D. Implications for *dc* transport

As we demonstrated in the previous subsection, the parametric instability that develops in the case of two *ac* fields above a certain threshold, results in the component of the momentum, $\mathcal{P}_3\exp\left[i\left(\omega_1+\omega_2\right)t/2\right]$, which well above the threshold has the magnitude $|\mathcal{P}_3|\approx\sqrt{|\mathcal{P}_1||\mathcal{P}_2|}$. The important consequence of the developed parametric resonance is, that the component $\mathcal{P}_3$ gives rise to new harmonics in the term $\propto|\mathcal{P}|^2\mathcal{P}$ in the equation of motion Eq. (2). Of particular interest are the $\mathcal{P}_3$-induced terms

$$\left[\mathcal{P}_1^2\mathcal{P}_3^*+\mathcal{P}_1\mathcal{P}_2^*\mathcal{P}_3\right]\exp\left[i\frac{3\omega_1-\omega_2}{2}t\right]. \tag{36}$$

It is easy to see that, under the condition $\omega_2\approx3\omega_1$, these terms act as an effective *dc* field, and thus generate low-frequency *circular* current even without *dc* drive. If the relation between the frequencies is precisely $1:3$, then the magnetic field, at which the developed parametric instability would give rise to a quasistationary circular current distribution, can be determined from Eq. (25)

$$\omega_c\simeq2\omega_1\left[1+\frac{2e^2\left(\mathcal{E}_1^2+\mathcal{E}_2^2\right)}{mE_0\left(\omega_1-\omega_2\right)^2}\right]. \tag{37}$$

If the ratio ω_2/ω_1 is close, but not exactly $1:3$, there is still a certain allowance, determined by Eq. (28) for the formation of the quasistationary current. The above effect of spontaneous formation of *dc*-like currents under irradiation is distinctively different from the formation of current domains when σ_d turns negative under irradiation. Firstly, the effect is specific to bichromatic irradiation. Secondly, it requires rather strict commensurability between the two frequencies, and finally, it develops within a very narrow interval around a certain magnetic field.

IV. CONCLUSIONS

In the present paper we have considered the problem of single electron motion in a magnetic field under irradiation by two monochromatic fields. When the frequencies ω_1 and ω_2 differ only slightly, $|\omega_1-\omega_2|\sim1/\tau$, the effect of a weak

nonparabolicity of the electron spectrum on the diagonal conductivity is qualitatively the same for monochromatic[27] and bichromatic irradiation. The prime qualitative effect which distinguishes the bichromatic case is the emergence of a parametric resonance at magnetic fields $\omega_c = (\omega_1 + \omega_2)/2$ and $\omega_c = |\omega_1 - \omega_2|/2$ when the detuning is strong (of the order of the cyclotron frequency). It is instructive to compare this effect with the parametric resonance of electrons in a magnetic field due to a weak time modulation of the field amplitude.[29,30,31] The latter effect, considered more than 20 years ago, has a transparent explanation. The modulation of the magnitude of a *dc* field with frequency $2\omega_c$ translates into a corresponding modulation of the cyclotron frequency, so that the equation of motion of the electron reduces to that for a harmonic oscillator with a weakly time-modulated eigenfrequency. The solution of this equation is unstable, if the modulation frequency is close to $2\omega_c$. As a natural stabilizing mechanism of the parametric resonance, the authors of Ref. 29 considered the nonparabolicity Eq. (1) of the electron dispersion. For a characteristic magnetic field of $B = 0.1$ T, the cyclotron frequency is $\omega_c = 3.6 \cdot 10^{11}$ Hz, i.e. in the microwave range so that conventional modulation of B with a frequency $2\omega_c$ is technically impossible. To bypass this obstacle, it was proposed in Ref. 31 to use microwave illumination with frequency $2\omega_c$ to create a parametric resonance. The idea was that the *magnetic field* component of the pumping electromagnetic wave would provide the necessary oscillatory correction to the external *dc* magnetic field. In the present paper, we have demonstrated that two nonresonant *ac* sources can *enforce* a parametric resonance of the type considered in Refs. 29,30,31 *without* any time modulation of the *dc* magnetic field. Remarkably, this bichromatic-radiation-induced cyclotron resonance emerges due to the same nonparabolicity Eq. (1) that played the role of a stabilizing factor in Refs. 29,30,31. Roughly, the time modulation of ω_c in the *dc* field required in Refs. 29,30,31 for parametric resonance emerges from the "beatings" of the responses to the two *ac* signals. The nonparabolicity transforms these beatings into a modulation of the cyclotron frequency. Although the increment, Γ, for parametric resonance, induced by bichromatic microwave irradiation, is proportional to the product $\mathcal{E}_1\mathcal{E}_2$ of the amplitudes of the two sources, while in Ref. 31 it was proportional to the *first power* of the magnetic component of the *ac* field, the "bichromatic" increment is much bigger. As demonstrated above, the bichromatic increment is $\Gamma \sim \omega_c \left(mc^2/E_0\right)\left(\mathcal{E}_1\mathcal{E}_2/B^2\right)$, which should be compared to the increment $\Gamma \sim \omega_c\left(\mathcal{E}/B\right)$ of Ref. 31. The ratio contains a small factor $(\mathcal{E}/B)$ which is offset by the huge factor $\left(mc^2/E_0\right)$.

Acknowledgments

Two of the authors (MER and FvO) acknowledge the hospitality of the Weizmann Institute of Science (supported by the Einstein Center and LSF grant HPRI-CT-2001-00114) while some of this work was performed. This work was also supported by the NSF-DAAD Collaborative Research Grant No. 0231010 and the DFG-Schwerpunkt "Quanten-Hall-Systeme."

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